Handout 1 - ECON703 (Fall 2023)

1 General Information

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Instructor:	Prof. John Kennan	jkennan@ssc.wisc.edu	`users.ssc.wisc.edu/~jkennan/`	SOC SCI 6434
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	Time	**Days**	**Room**
Lectures:	04:30 PM to 06:45 PM	Weekdays Aug 16th - Aug 31st	SOC SCI 6102
Office Hours:	01:30 PM to 03:30 PM	Mondays	SOC SCI 6434
TA Sessions:	09:55 AM to 10:45 AM 11:00 AM to 11:50 AM	Weekdays Aug 17th - Aug 31st	SOC SCI 6102
TA Office Hours:	12:30 PM to 2:30 PM	Weekdays Aug 17th - Aug 31st	SOC SCI 7218

Miscellaneous:

- Material will be available on Canvas and handouts (additionally) on my website.
- I will upload my notes. Try to focus on following the session and asking questions over taking notes.
- You can also email me if you have questions outside of office hours.
- `https://www.wolframalpha.com/` will be very useful.
- Useful inequalities cheat-sheet: `https://www.lkozma.net/inequalities_cheat_sheet/ineq.pdf`

2 Notation

Quantors give us information about *how many* objects have a property.

- $\forall$: **for all** ($\forall a \in A$ means for every element a in $A \dots$)
- $\not\forall$: **not for all** ($\not\forall a \in A$ means not for every element a in $A \dots$)
- $\exists$: **there exists** ($\exists a \in A$ means that there is at least one element a in $A \dots$)
- $\nexists$: **there does not exist** ($\nexists a \in A$ means that there is no element a in $A \dots$)
- $\exists!$: **there exists exactly one** ($\exists! a \in A$ means that there is exactly one element a in $A \dots$)

Another symbol we often use is $\neg$ (**negation**): $\forall a \in A : \neg p(a)$ (where $p(a)$ is a statement on a and $\neg p(a)$ is its negation.) Additionally, we often want to say something about a combination of statements. For this purpose, let $p(\cdot)$ and $q(\cdot)$ be two statements.

- $\wedge$: **logical and** ($\forall a \in A : p(a) \wedge q(a)$ means that for all elements a in A, both $p(a)$ and $q(a)$ are true.)
- $\vee$: **logical or** ($\forall a \in A : p(a) \vee q(a)$ means that for all elements a in A, $p(a)$ or $q(a)$ or both are true.)
- $\oplus$: **exclusive logical or** ($\forall a \in A : p(a) \oplus q(a)$ means that for all elements a in A, either $p(a)$ or $q(a)$ is true.)
- $\implies$: **implies** ($p(a) \implies q(a)$ means that if statement p is true for a, then q is necessarily true for a.)
- $\iff$: **is equivalent to** ($p(a) \iff q(a)$ means that either both p and q are true for a or neither.)

3 Proofs

Example 3.1 (Direct Proof) The square of an even number is even.
Assume that n is an even integer. Then, by definition, it can be written as $n = 2a$ where a is an integer.

$$n^2 = (2a)^2 = 4a^2 = 2(2a^2)$$

Thus, the square of n can be written as two times the integer $2a^2$. Hence, it is an even number.

Theorem 3.1 (Contrapositive)

$$(p(a) \implies q(a)) \iff (\neg q(a) \implies \neg p(a))$$

In English, this means that the following two statements are equivalent. I hope you forgive the blatantly incorrect statement about waterfowl.

- *If something is a swan, it is white.*
- *If something is not white, it is not a swan.*

Sometimes, it is easier to prove the contrapositive of a statement than the statement itself.

Example 3.2 (Proof by Contradiction - 1) There are no boring Natural Numbers
Assume (for the sake of contradiction) that there are boring natural numbers. Let $B \subset \mathbb{N}$ be the set of boring numbers. Then there is smallest boring natural number $\underline{b} = \min(B)$. But clearly, being the smallest boring natural number is an interesting property. Thus, $\underline{b} \notin B$.
Contradiction!
Therefore, there are no boring natural numbers.

Example 3.3 (Proof by Contradiction - 2) Some Set Manipulation
Let A and B be some arbitrary sets. We want to show the following statement:

$$A \cap (B \backslash A) = \emptyset$$

Assume (for the sake of contradiction) that $x \in A \cap (B \backslash A)$. Then $x \in A$ and $x \in B \backslash A$. But

$$(x \in B \backslash A) \iff (x \in B \wedge x \notin A)$$

And thus,

$$x \in A \wedge x \notin A$$

Contradiction!
Therefore, the statement holds.

Example 3.4 (Proof by Induction - 1) The Young Carl-Friedrich Gauss
We want to show that the sum of the natural numbers up to n is given by

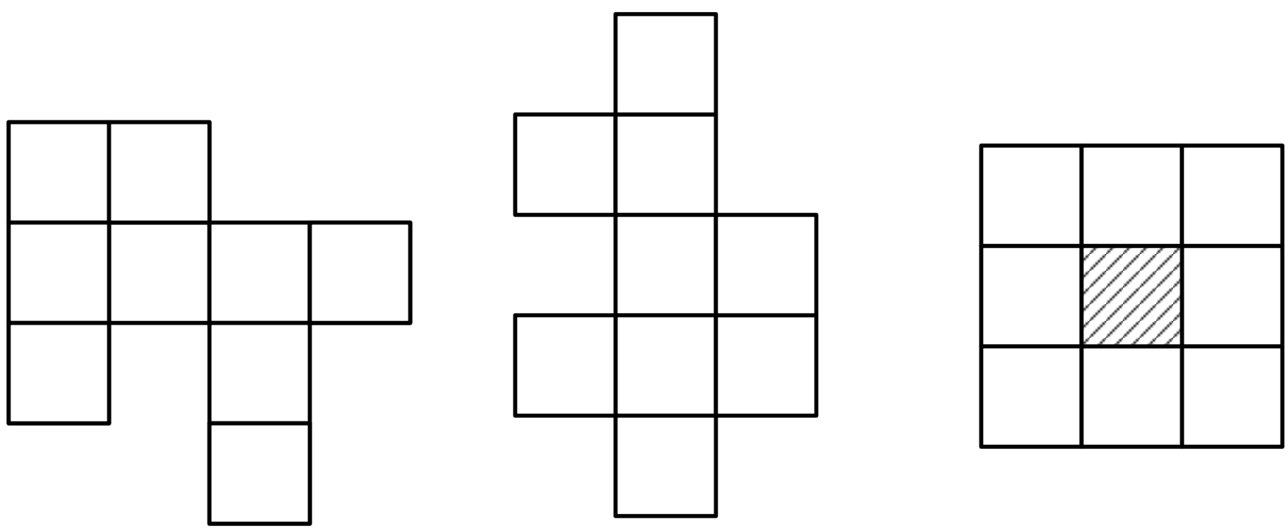
$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

We start our induction at $n = 1$. Clearly $\sum_{i=1}^{1} i = 1 = \frac{1(1+1)}{2}$. So our property holds for $n = 1$.
Next, we show the induction step. Assume that our postulated formula holds for some n^*. Then we can show the induction step as follows.

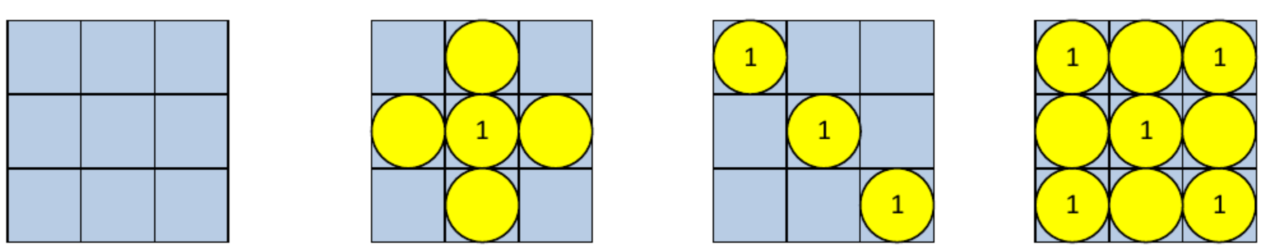
$$\sum_{i=1}^{n^*+1} i = (n^*+1) + \sum_{i=1}^{n^*} i = (n^*+1) + \frac{n^*(n^*+1)}{2} = \frac{(n^*+1)(n^*+2)}{2}$$

Thus, we know the following. The statement holds for $n = 1$ since we showed it does in the base case. Therefore, it holds for $n = 2$ by the induction step. Therefore, it holds for $n = 3$ by the induction step. And so on and so forth.

Example 3.5 (Proof by Induction - 2) The lamp game is solvable for every setup
Imagine a series of square boxes, each of which contains a switch and a lightbulb. The boxes are laid out on a grid so that each box shares at least one wall with another box. However, the overall setup does not have to be square or regular. For example, the following layouts are valid:



If you activate the switch in one box, the state of the lamp (on or off) in the box itself and all adjacent boxes changes.



Is it possible to switch all lamps on? If so, is it possible for each setup of boxes?